TRINITY COLLEGE

Mathematics Applications Units 3, 4
Test 5 2018

Calculator Free

Spanning Trees and Maximum Flow

STUDENT'S NAME _____

DATE: Thursday 16 August **TIME:** 50 minutes **MARKS**: 48

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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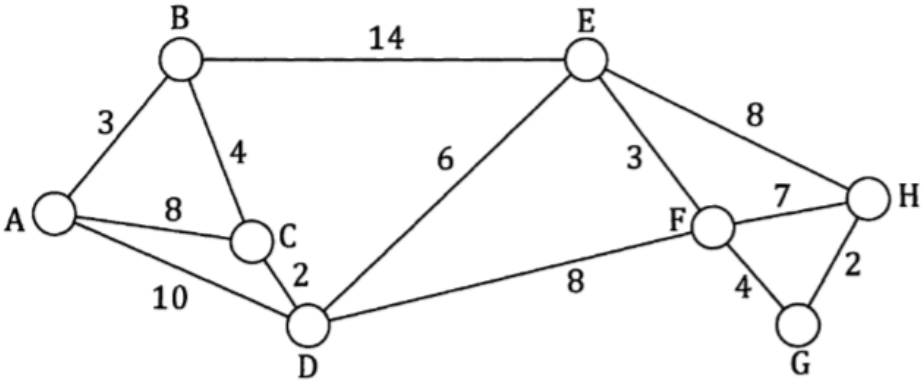
1. (6 marks)

The following weighted graph gives the distances, in km, between villages along the roads shown.



(a) Find the route of the shortest path(s) from village A to village H. [3]

(b) What is the length of this shortest route? [1]

(c) The road between villages E and F is closed due to roadworks. What effect, if any, does this have on the answers to (a) and (b) above. [2]

2. (5 marks)

A water treatment plant has six storage tanks which need to be connected by pipelines. The estimated cost, in thousands of dollars, of installing a pipeline between any two tanks is given. The dash entries indicate that a pipeline cannot be installed.

	A	**B**	**C**	**D**	**E**	**F**
A	-	7	-	-	8	5
B	7	-	11	-	6	-
C	-	11	-	6	4	-
D	-	-	6	-	10	7
E	8	6	4	10	-	-
F	5	-	-	7	-	-

(a) Draw the minimal cost spanning tree. [3]

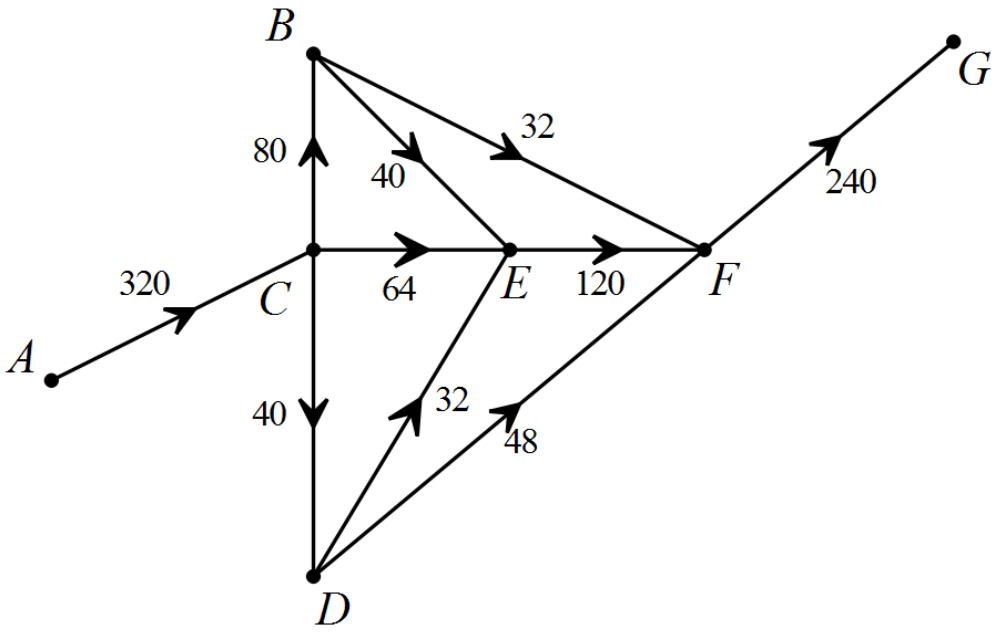
(b) Calculate the estimated minimum cost of connecting these tanks. [1]

(c) It was discovered that the connection between tanks A and E could not proceed. What effect, if any, would this have on the estimated minimum cost? Explain. [2]

3. (9 marks)

1. [4, 1, 1, 1 = 7 marks]

Each arc on the diagram below represents the maximum amount of water through each pipe in kilolitres per hour.



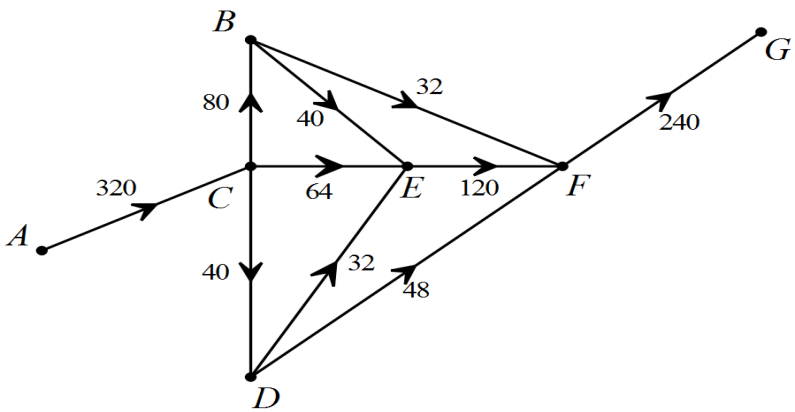
(a) Calculate the maximum flow.

(b) Show, on the diagram above, the minimum cut that validates your answer in part (a).

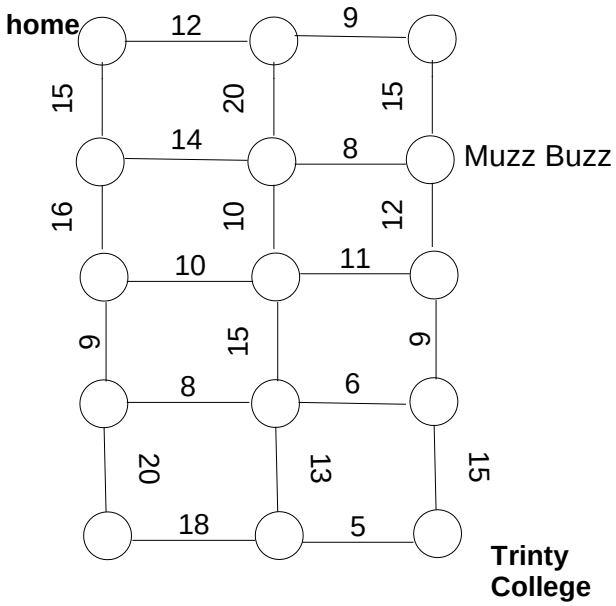
(c) Describe the effect on the maximum flow if:

(i) Pipe BE was removed.

(ii) Pipe EF was reduced to 75.



There are several alternate routes from Mr Presser's home to Trinity College. Distance is not a factor however, time delays at traffic lights and stop signs create the most annoyance. Out of frustration Mr Presser decides to time the delays in order to find the least frustrating way to drive to Trinity College. The time delays, in seconds, that he encounters on different routes are recorded on the network below. The paths through the network move either right or down.

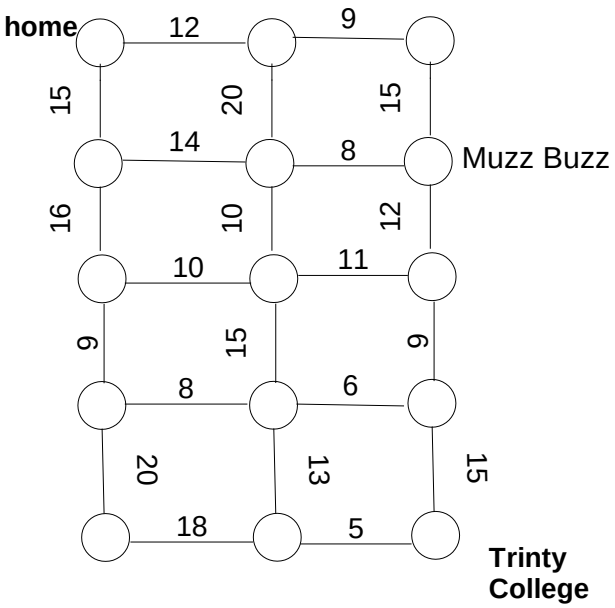


(a) On the network above, show the route that has the smallest total time delay from home to Trinity College.

(b) What is this total smallest time delay?

(c) To travel between each traffic light or stop sign it takes approximately 1 minute 15 seconds. On a particular day Mr Presser stops at Muzz Buzz for coffee and this takes 135 seconds.

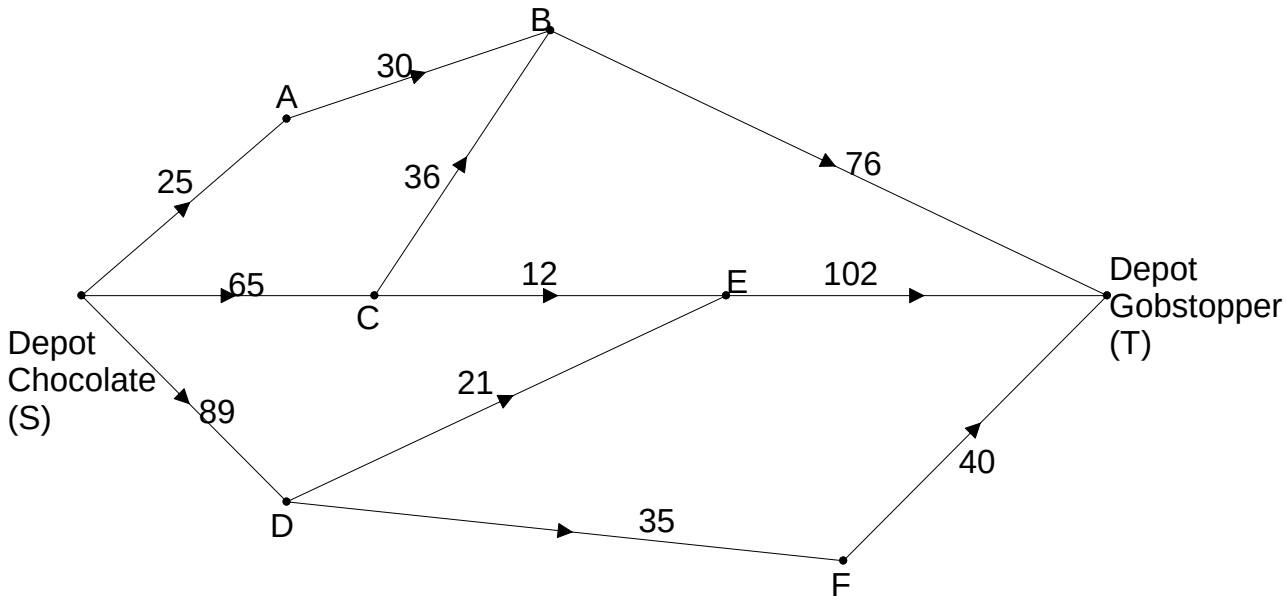
(i) Show, on the second network below, the new route with the smallest time delay that Mr Presser should take to school if he stops at Muzz Buzz.



(ii) What is this new total smallest time delay now?

(iii) Did Mr Presser make a meeting that was scheduled for 7.30 am if he left home at 7.15 am? (Show all working to support your answer).

The Oompa Loompas are busy at work in Willy Wonka's Chocolate Factory hauling kilograms of confectionery. The network below represents the "behind the scenes" Chocolate Factory paths and the amount of confectionery in kilograms that can be transported from Depot Chocolate (S) to Depot Gobstopper (T) per hour. Each of the nodes represents a chocolate refuelling junction.

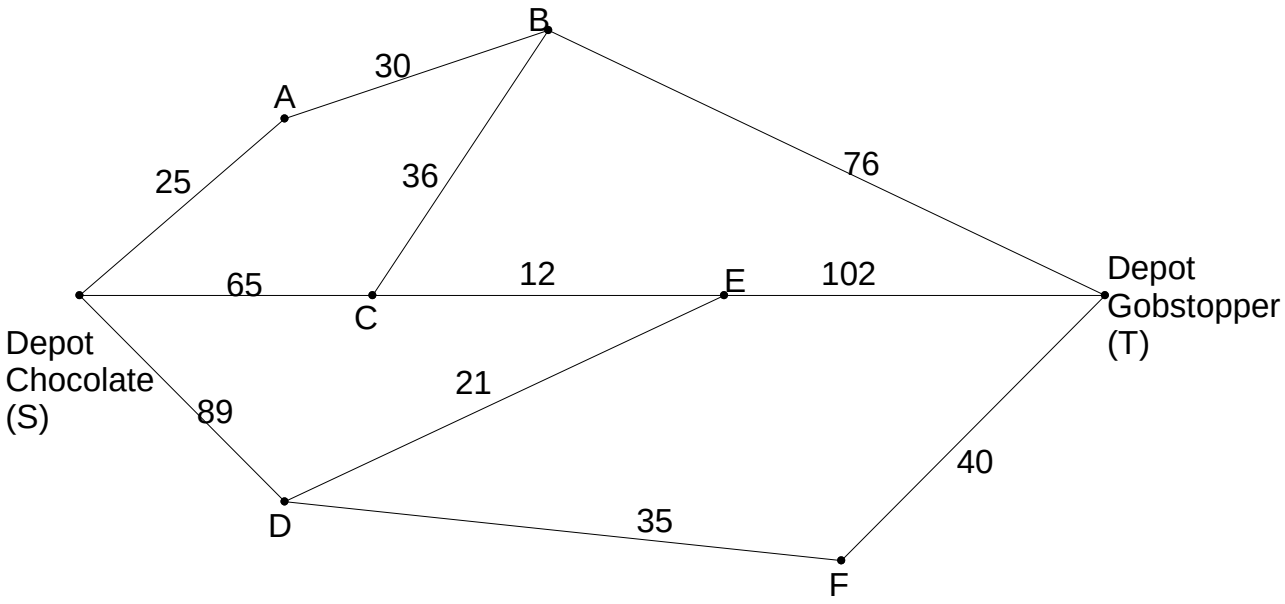


(a) Determine the maximum number of kilograms of confectionery that can be transported by the Oompa Loompas each hour from S to T.

(b) Augustus Gloop drinks too much chocolate and gets sucked into path DF which consequently blocks the flow of confectionery transportation along this route. By how much does this closure of arc DF affect the maximum flow?

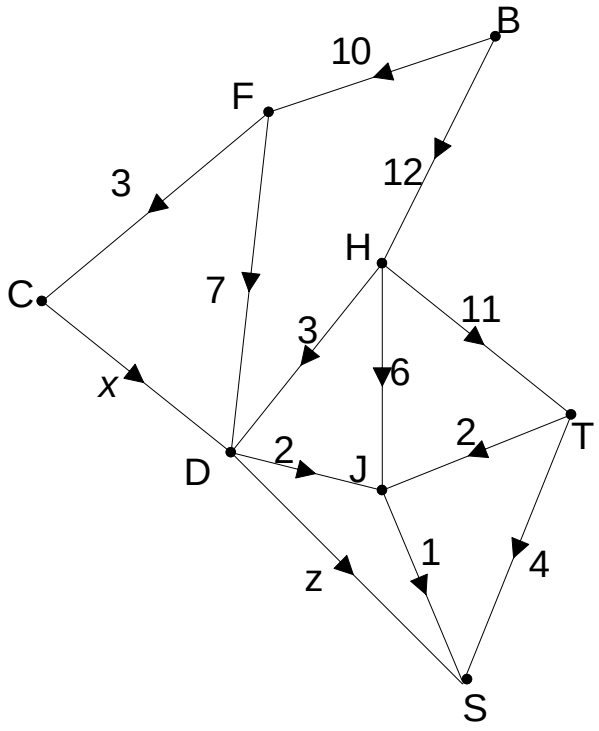
(c) Willy Wonka wants to improve the transportation of confectionery by assigning extra Oompa Loompas to just one arc. (Using the original network)Which arc should he choose in order to maximise the flow and by how much will this improve the maximum flow?

Willy Wonka's security system involves lining each of the arcs between the depots with special chocolates. The network below represents the number of chocolates needed per arc.



(d) If each depot is connected to each other depot (not necessarily directly), determine the minimum number of chocolates needed for Willy Wonka's security system. Clearly indicate on the above network which arcs will be lined with special chocolates as part of the security system.

The shortest path from B to S in the network below is B – F – C – D – J – S.
All values are integers.



(a) Determine the possible positive integer value(s) of x ($x > 0$) which will achieve the shortest path B – F – C – D – J – S.

(b) For the value(s) of x chosen in part (a), determine the minimum integer value that z can take to maintain the shortest path outline in part (a).

(a) Draw a network diagram showing the links and times for the information shown in the table below.

Travelling Times (in hours)

	A	**B**	**C**	**D**	**E**	**F**	**G**
A	-	-	-	-	8	3	-
B	-	-	1	-	7	-	-
C	-	1	-	-	6	4	2
D	-	-	-	-	9	-	-
E	8	7	6	9	-	-	-
F	3	-	4	-	-	-	5
G	-	-	2	-	-	5	-

(b) Find the fastest route to get from A to B and state the minimum time.